Math 1A: Calculus **Instructor: Alexander Paulin**

Handout: More Differentiation Practice

Discussions 201, 203 // 2018-10-05

Problem 1. Compute the derivative of $e^{-1/x}$.

Problem 2. Expand $\frac{d^2}{dx^2} f(g(x))$.

Problem 3. Compute the derivative of

$$f(x) = \sin(\sqrt{x^2+1} + 3x).$$

Problem 4. The supply $S(t)$, demand $D(t)$, and price $P(t)$ for some good are all functions of time. They are related as follows:

$$D(t) = 100 - 5P(t),$$
$$S(t) = 3P(t) - 20.$$

How are $D'(t)$ and $S'(t)$ related?

Problem 5. Compute the derivative of *hyperbolic tangent*, which is defined as

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

Problem 6 (Stewart §3.4 #95)**.** Let n be a positive integer. Prove that

$$\frac{d}{dx}(\sin^n(x)\cos(nx)) = n\sin^{n-1}(x)\cos((n+1)x).$$

Problem 7. Compute

$$\frac{d^{12}}{dx^{12}}\left[x^{11} + (x^4 + 5x^3 + 6x + 3)^3 + e^{-x} + \sin(2x)\right].$$

(Hint: there are effectively four parts to this problem; it may be best to think about them separately.)

Problem 8. Consider the function

$$f(x) = \begin{cases} x^2\sin(1/x) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

(1) Find the derivative $f'(x)$. (You calculated $f'(0)$ on the previous homework.)
(2) Determine the limit

$$\lim_{x\to 0} f'(x).$$

Compare to the value of $f'(0)$.

Then answer the following true/false questions.

________**:** f is defined everywhere.
________**:** f is continuous everywhere.
________**:** f is differentiable everywhere.
________**:** f' is defined everywhere.
________**:** f' is continuous everywhere.
________**:** f' is differentiable everywhere.

Problem 9. Differentiate both sides of the identity

$$\tan(\arctan(x)) = x,$$

using the chain rule on the left. Use this to find a formula for the derivative of $\arctan(x)$. Does the answer surprise you? Strategies analogous to this one can be used to compute the derivatives of various inverse functions.

Problem 10. It is a fact that the derivative of $\ln x$ is $1/x$, as you will see in lecture (or as you can check yourself in the manner of the preceding problem). Find a way to use this information to compute the derivative of the function

$$f(x) = x^x.$$